BRIEF COMMUNICATIONS

EXTREME MODES IN THE SYSTEM OF DIFFERENTIAL HEAT- AND MASS-TRANSFER EQUATIONS

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With regard to the desiccation process and the experimental methods of determining the thermophysical characteristics of capillary-porous colloidal bodies, the system of differential heat- and mass-transfer equations is analyzed for extreme modes.

The system of heat- and mass-transfer equations

$$\frac{\partial u}{\partial \tau} = K_{11}\nabla^2 u + K_{12}\nabla^2 T, \tag{1}$$

$$\frac{\partial T}{\partial \tau} = K_{22}\nabla^2 T + K_{21}\nabla^2 u \tag{2}$$

describes the molecular (diffusive) transfer of mass (moisture) and heat through capillary-porous bodies. It is assumed in the derivation of Eqs. (1) and (2) that the coefficients of heat and mass transfer (λ, a_m, a_m^T, a) as well as the thermophysical properties (c, r, ρ_0, δ) are independent of the coordinates. Furthermore, the temperature of moisture in the capillaries is considered equal to the temperature of capillary walls throughout the entire process of heat and mass transfer, which is true only in the case of diffusive transfer (convective transfer, including filtration, is disregarded here). Coefficients K_{ij} ($i, j = 1, 2$) are not subject to the mutuality principle, i.e., $K_{ij} \neq K_{ji}$ and for liquid–vapor moisture [1]

$$K_{11} = a_m; \quad K_{12} = a_m^T = a_m\delta, \tag{3}$$

$$K_{22} = a + a_{m1}^T \frac{r_{12}}{c}; \quad K_{21} = a_{m1}\frac{r_{12}}{c}. \tag{4}$$

Equations (1) and (2) describe the transient as well as the steady heat and mass transfer. In the case of desiccation (transient heat and moisture transfer), the second term on the right-hand side of (2) may be replaced by $(\varepsilon r_{12}/c)(\partial u/\partial \tau)$, because for moisture sources in the vapor–liquid system we have

$$I_{12} = -I_{21} = \varepsilon\rho_0 \frac{\partial u}{\partial \tau} = \operatorname{div} \rho_0 (a_{m1}\nabla u + a_{m1}^T \nabla T). \tag{5}$$

The system of equations (1), (2) is retained here, only coefficients K_{22} and K_{21} become

$$K_{22} = a + a_{m1}^T \frac{r_{12}}{c} = a + \frac{\varepsilon r_{12}}{c} a_m\delta, \tag{4a}$$

$$K_{21} = a_{m1}\frac{r_{12}}{c} = \frac{r_{12}\varepsilon}{c} a_m. \tag{4b}$$

We will now consider the extreme modes.

1) The temperature of the moist capillary-porous body will be assumed constant throughout the process of heat and mass transfer: $\partial T/\partial \tau = 0$. There are two possibilities then: a) from Eq. (2) follows $\nabla^2 T = 0$ and $\nabla^2 u = 0$. This is a trivial case of equilibrium (the temperature and the moisture content not only do not vary with the time but are also independent of the space coordinates: u = const and T = const). b) Since

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$T \equiv T(x_i, \tau)$ and $u = u(x_i, \tau)$, hence from (2) with $\partial T/\partial \tau = 0$ follows

$$\nabla^2 u = -\frac{K_{22}}{K_{21}} \nabla^2 T \neq 0; \quad \left.\begin{matrix} u \\ T \end{matrix}\right\} = f(x_i), \tag{6}$$

i.e., the moisture field and the temperature field are similar. Inasmuch as the form of the distribution function $f(x_i)$ does not depend on time, the local velocity $\partial u/\partial \tau$ will, in accordance with Eq. (1), be also independent of time.

In the special case

$$\frac{\partial u}{\partial \tau} = \text{const} \tag{7}$$

the distribution of temperature and moisture in one-dimensional problems is described by a simple parabola. This is the case during the constant-rate period of the desiccation process. The moisture content at any point is a linear function of time ($\partial u/\partial \tau = \text{const}$), while the moisture profile across the body thickness (infinitely large plate, cylinder, sphere) is described by a parabola. The temperature at any point does not vary with time ($\partial T/\partial \tau = 0$). If during the constant-rate desiccation period there occurs evaporation inside the body ($\varepsilon \neq 0$, $a_{m1} \neq 0$), then, according to (6) with $K_{21} \neq 0$, the temperature profile in such one-dimensional problems is described by a parabola. c) In the special case without evaporation of moisture inside the body during the constant-rate desiccation period ($\varepsilon = 0$ or $a_{m1} = 0$) the temperature is the same at all points of the body and, to the first approximation, equal to the wet-bulb temperature T_M ($T = T_M = \text{const}$, $\nabla^2 T = 0$). This does not contradict relation (6), because $\varepsilon = 0$ and $K_{21} = 0$ but $K_{22} \neq 0$ and $\nabla^2 u \neq 0$ for $\nabla^2 T = 0$. Indeed, for $\partial T/\partial \tau = 0$, from (2) rewritten as

$$\frac{\partial T}{\partial \tau} = a\nabla^2 T + \frac{\varepsilon r_{12}}{c} \frac{\partial u}{\partial \tau},$$

follows that $\nabla^2 T = 0$ when $\varepsilon = 0$.

From here Eq. (1) yields the classical equation of diffusion

$$\frac{\partial u}{\partial \tau} = K_{11} \nabla^2 u, \tag{1a}$$

indicating that the moisture transfer during the constant-rate period of desiccation is an isothermal process.

At a constant desiccation rate ($d\bar{u}/d\tau = \text{const}$) the local rate $\partial u/\partial \tau$ is also constant. It then follows from (1a) that

$$\nabla^2 u = \text{const} \quad \text{at} \quad \frac{\partial u}{\partial \tau} = \text{const}. \tag{6a}$$

This is noted during the desiccation of slowly drying colloidal materials as, for example, gelatin.

All these modes of moisture and temperature distribution occur during the constant-rate period of desiccation in capillary-porous colloidal bodies and, as of now, are deduced from experimental evidence and empirical laws.

2) The moisture content in a body will now be assumed constant throughout the process of heat and mass transfer (quasisteady state during the heating of a moist body), i.e., $\partial u/\partial \tau = 0$. It then follows from (1) that

$$\nabla^2 u = -\frac{K_{12}}{K_{11}} \nabla^2 T \quad \text{at} \quad \frac{\partial u}{\partial \tau} = 0. \tag{8}$$

If $K_{11} = a_m \neq 0$ and $K_{12} = a_m^T \neq 0$, then the moisture field is similar to the temperature field. At $\partial u/\partial \tau = 0$, however, the local rate $\partial T/\partial \tau \neq 0$, in accordance with Eq. (2), i.e., the temperature profile inside the body does not vary with time, because the temperature at all points in the body varies with time according to the same law. Such situations are encountered in experimental thermophysics as, for example, during the quasisteady heating of a moist body for the determination of the temperature-gradient coefficient, the thermal diffusivity, and the thermal conductivity. In this method the moist body is heated at a constant rate (the ambient temperature rises linearly with time). From some instant on, the temperature at any point in the body becomes a linear function of time and the temperature profile in one-dimensional cases with symmetry is described by a parabola [2]. The moisture profile corresponding to a parabolic temperature profile is also parabolic. From the drops in temperature and moisture content one determines coefficients a and δ, whereupon, if the heat-transfer coefficient is known, the thermal conductivity can also be determined [3]. It is entirely plausible that a trivial situation may occur in case (2) with $\partial u/\partial\tau = 0$, namely $\partial T/\partial\tau = 0$ when $\nabla^2 u = 0$ and $\nabla^2 T = 0$ (steady or equilibrium state).

In conclusion, we will consider Eqs. (1) and (2) becoming the equation of heat conduction, their becoming the equation of diffusion having been discussed earlier. In a perfectly dry body there occurs no mass transfer ($K_{11} = K_{12} = K_{21} = 0$ at $u = 0$) and from (1), (2) there follows the equation of heat conduction*

$$\frac{\partial T}{\partial \tau} = K_{22}\nabla^2 T = a\nabla^2 T. \qquad (9)$$

The same equation will be obtained from (1), (2) for a moist body with the maximum possible moisture content u_{max} (distension moisture). At moderate temperature drops there will be no moisture transfer then ($K_{11} = K_{12} = K_{21} = 0$ when $u = u_{max}$) and, consequently, $\partial u/\partial\tau = 0$ with Eq. (9) following from (1) and (2). In this case the moist body heats up like a dry body, namely without a redistribution and evaporation of moisture (the moisture content at any point in the body remains constant and equal to u_{max}). The system of equations of heat and mass transfer under a pressure gradient ($\nabla P \neq 0$) can be analyzed in an analogous manner. The earlier results remain valid here too. It must be assumed here that the moisture and the matrix body are at the same temperature and that Darcy's law (convective diffusion) applies. Other assumption concerning the thermophysical properties are also retained.

It is noted, finally, that only an analysis of the solution to system (1), (2) has made it possible not only to thoroughly explain the mechanism of heat and mass transfer during desiccation of diverse materials but also to develop several quick methods of measuring the thermophysical properties of moist capillary-porous materials [4].

NOTATION

$u \equiv u(x_i, \tau)$	is the moisture content in a body;
$\bar{u} \equiv \bar{u}(\tau)$	is the mean moisture content;
$T \equiv T(x_i, \tau)$	is the temperature of a body;
τ	is the time;
x_i	is the Cartesian coordinates ($i = 1, 2, 3$: $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$);
a	is the thermal diffusivity;
λ	is the thermal conductivity;
c	is the specific heat of moist material;
ρ_0	is the density of perfectly dry material;
$a_m = a_{m1} + a_{m2}$	is the moisture diffusivity;
a_{m1}	is the diffusivity of vapor moisture;
a_{m2}	is the diffusivity of liquid moisture;
r_{12}	is the specific heat of liquid evaporation or of vapor condensation ($r_{12} = r_{21}$);
a_m^T	is the moisture thermodiffusivity;
δ	is the temperature-gradient coefficient.

LITERATURE CITED

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*It follows from the equality $u = 0$ with coefficient $K_{12} = 0$, according to Eq. (1), $\nabla^2 T = 0$.